Mathematics Scope and Sequence – High School
Geometry

Standards

NCTM Principles and Standards

The National Council of Teachers of Mathematics (NCTM) created its ***Principles and Standards for School Mathematics*** in 2004. From the introduction to Grades 9-12: "These Standards describe an ambitious foundation of mathematical ideas and applications intended for all students. Through its emphasis on fundamental mathematical concepts and essential skills, this foundation would give all students solid preparation for work and citizenship, positive mathematical dispositions, and the conceptual basis for further study… Through their high school experiences, they stand to develop deeper understandings of the fundamental mathematical concepts of function and relation, invariance, and transformation.

In high school, students should build on their prior knowledge, learning more-varied and more-sophisticated problem-solving techniques. They should increase their abilities to visualize, describe, and analyze situations in mathematical terms. They need to learn to use a wide range of explicitly- and recursively-defined functions to model the world around them. Moreover, their understanding of the properties of those functions will give them insights into the phenomena being modeled.

Secondary school students need to develop increased abilities in justifying claims, proving conjectures, and using symbols in reasoning. They can be expected to learn to provide carefully reasoned arguments in support of their claims. They can practice making and interpreting oral and written claims so that they can communicate effectively while working with others and can convey the results of their work with clarity and power. They should continue to develop facility with such technological tools as spreadsheets, data-gathering devices, computer algebra systems, and graphing utilities that enable them to solve problems that would require large amounts of computational time if done by hand.

A central theme of *Principles and Standards for School Mathematics* is connections. Students develop a much richer understanding of mathematics and its applications when they can view the same phenomena from multiple mathematical perspectives. One way to have students see mathematics in this way is to use instructional materials that are intentionally designed to weave together different content strands. Another means of achieving content integration is to make sure that courses oriented toward any particular content area (such as algebra or geometry) contain many integrative problems—problems that draw on a variety of aspects of mathematics, that are solvable using a variety of methods, and that students can access in different ways.

These Standards are demanding. It will take time, patience, and skill to implement the vision they represent… Such efforts are essential. We owe our children no less than a high degree of quantitative literacy and mathematical knowledge that prepares them for citizenship, work, and further study."

Geometry Expectations

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Students analyze properties and determine attributes of two- and three-dimensional objects;
explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them. They establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others. Students use trigonometric relationships to determine lengths and angle measures.

Specify locations and describe spatial relationships using coordinate geometry and other representational systems

Students use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations. They investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

Apply transformations and use symmetry to analyze mathematical situations

Students understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices. They use various representations to help understand the effects of simple transformations and their compositions.

Use visualization, spatial reasoning, and geometric modeling to solve problems

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Mathematics Scope and Sequence – High School
Geometry

Students draw and construct representations of two- and three-dimensional geometric objects using a variety of tools. They visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections and use vertex-edge graphs to model and solve problems. Students use geometric models to gain insights into, and answer questions in, other areas of mathematics. They use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Academic Language

Geometry Mathematical Language

Math vocabulary which students in this course are expected to master and use: http://www.emsc.nysed.gov/3-8/LanguageGeometry.doc

Commencement-Level Glossary of Mathematical Terms

Definitions of mathematical terms used in high school: http://www.emsc.nysed.gov/3-8/glossaryHS.doc

Lesson Planning

There are three daily planning guides available, each keyed to one of the recommended Geometry curricula for this grade:
Discovering Geometry (Key Curriculum Press) http://schools.nycenet.edu/offices/teachlearn/mathematics/rs/Discovering_Geometry_Pacing_Guide.doc
McDougal Littell *Geometry* http://schools.nycenet.edu/offices/teachlearn/mathematics/rs/McDougal_Geometry_Pacing_Guide.pdf
Prentice Hall *Geometry* http://schools.nyc.gov/Academics/Mathematics/EducatorResources/PrenticeHallGeometry.htm

Development of Content Topics and Concepts

For each content strand these documents trace the development year by year of every band (i.e., sub-skill) from elementary school through high school. As such they provide insight into the foundational work done in previous grades and map the subsequent elaboration of the topics in later courses.
Band Traces:

Number Sense and Operations: http://schools.nyc.gov/NR/rdonlyres/4C837772-3E02-4552-A02E-AE7E9AE769C1/47718/StrandTraceNumberSenseandOperations.pdf

Algebra: http://schools.nyc.gov/NR/rdonlyres/4C837772-3E02-4552-A02E-AE7E9AE769C1/47719/StrandTraceAlgebra.pdf

Geometry: http://schools.nyc.gov/NR/rdonlyres/4C837772-3E02-4552-A02E-AE7E9AE769C1/47720/StrandTraceGeometry.pdf

Measurement: http://schools.nyc.gov/NR/rdonlyres/4C837772-3E02-4552-A02E-AE7E9AE769C1/47721/StrandTraceMeasurement.pdf

Statistics and Probability: http://schools.nyc.gov/NR/rdonlyres/4C837772-3E02-4552-A02E-AE7E9AE769C1/47722/StrandTraceStatisticsandProbability.pdf

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Mathematics Scope and Sequence – High School
Geometry

Development of Content Topics and Concepts (continued)

The chart below depicts the number of performance indicators for the content strands of each course. It's important to note that the numbers refer to the **new** performance indicators introduced in each course; performance indicators from previous courses (and grade levels) are assumed to have been incorporated into students' repertory of skills and abilities. For example the Geometry course makes extensive use of algebraic concepts and procedures learned in the previous course, in order to solve geometric problems. However no new algebra topics are introduced in the Geometry course.

Number of Performance Indicators for Each Course				
Content Strand	***Integrated Algebra***	***Geometry***	***Algebra 2 and Trigonometry***	***Total***
Number Sense and Operations	8	0	10	18
Algebra	45	0	77	122
Geometry	10	74	0	84
Measurement	3	0	2	5
Statistics and Probability	23	0	16	39
TOTAL	**89**	**74**	**105**	**268**

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Mathematics Scope and Sequence – High School
Geometry

Assessment

Approximate Percentage of Questions Assessing Each Band

The questions on the Geometry Regents Exam assess both the content and the process strands of New York State Mathematics Standard 3. However, since **all** performance indicators (i.e., topics) for this course are within the geometry content band, in this case it is more useful to categorize them into bands, which are a sub-category within each content strand. Each question is aligned to at least one content performance indicator, but is also aligned to one or more process performance indicators, as appropriate for the concepts embodied in the task. As a result of the alignment to both process and content strands, the tests assess students' conceptual understanding, procedural fluency, and problem-solving abilities rather than assessing knowledge of isolated skills and facts.

Content Band	% of Total Credits
Geometric Relationships	8–12%
Constructions	3–7%
Locus	4–8%
Informal and Formal Proofs	41–47%
Transformational Geometry	8–13%
Coordinate Geometry	23–28%

Previous Geometry Regents Exams can be downloaded at: http://www.jmap.org/JMAP_REGENTS_EXAMS.htm

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Mathematics Scope and Sequence – High School
Geometry

Calculator Policy

Listed below are the courses in grades 7-12 that require access to calculators, and the specific calculator(s) that may be used on the assessment. More than providing access on the day of the exam, schools must ensure that students possess facility in the use of these calculators. Therefore teachers should integrate their regular use as part of the year's coursework.

Course		4-Function Calculator		Scientific Calculator		Graphing Calculator
7th Grade Math	Required	no		yes		no
8th Grade Math	Required	no		yes		no
Math A	Required	no		yes	**OR**	yes
Math B	Required	no		no		yes
Integrated Algebra	Required	no		no		yes
Geometry	Required	no		no		yes
AP Calculus	Required	no		no		yes
AP Statistics	Required	no		no		yes
PSAT	Required	yes	**OR**	yes	**OR**	yes

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Mathematics Scope and Sequence – High School
Geometry

Selected Internet Websites - Geometry

These on-line resources can serve to reinforce student mathematical skills and concepts, and to help teachers differentiate their mathematics instruction.

STRAND	TOPIC	WEBSITE
Geometry	Properties of two- and three-dimensional shapes	http://mathworld.wolfram.com/Dimension.html http://mathforum.org/dr.math/faq/formulas/faq.figuredef.html http://nlvm.usu.edu/en/nav/category_g_3_t_3.html
Geometry	Coordinate Geometry	http://www.mathvids.com/lesson/mathhelp/176-coordinate-geometry http://www.wiziq.com/tutorial/177-Coordinate-Geometry
Geometry	Transformation Geometry	http://standards.nctm.org/document/eexamples/chap6/6.4/index.htm http://www.svsu.edu/emplibrary/Geometry%20Unit.ppt#256,1,"Transformations" High School Geometry

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Mathematics Scope and Sequence – High School
Geometry

September / October	October / November	December / January
Unit 1 - Introduction to Geometry: Lines, Angles, Measurement, Basis Constructions, Reasoning and Proof	***Unit 2 - Lines and Angles, Congruent Triangles***	***Unit 3 - Relationship within Triangles, Similarities***
Geometric Relationships G.G.1 Know and apply that if a line is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to the plane determined by them. ***Constructions*** G.G.17 Construct a bisector of a given angle, using a straightedge and compass, and justify the construction G.G.18 Construct the perpendicular bisector of a given segment, using a straightedge and compass, and justify the construction. ***Formal and Informal Proofs*** G.G.24 Determine the negation of a statement and establish its truth value. G.G.25 Know and apply the conditions under which a compound statement (conjunction, disjunction, conditional, biconditional) is true G.G.26 Identify and write the inverse, converse, and contrapositive of a given conditional statement and note the logical equivalences G.G.27 Write a proof arguing from a given hypothesis to a given conclusion	***Geometric Relationships*** G.G.8 Know and apply that if a plane intersects two parallel planes, then the intersection is two parallel lines ***Constructions*** G.G.20 Solve problem using compound loci G.G. 21 Graph and solve compound loci in the coordinate plane ***Locus*** G.G.19 Construct lines parallel (or perpendicular) to a given line through a given point, using a straightedge and compass, and justify the construction. ***Formal and Informal Proofs*** G.G.27 Write a proof arguing from a given hypothesis to a given conclusion G.G.28 Determine the congruence of two triangles by using one of the five congruence techniques (SSS, SAS, ASA, AAS, HL), given sufficient information about the sides and/or angles of two congruent triangles G.G.29 Identify corresponding parts of congruent triangles G.G.30 Investigate, justify, and apply theorems about the sum of the measures of the angles of a triangle G.G.31 Investigate, justify, and apply the isosceles triangle theorem and its converse	***Locus*** G.G.21 Investigate and apply the concurrence of medians, altitudes, angle bisectors, and perpendicular bisectors of triangles ***Formal and Informal Proofs*** G.G.24 Determine the negation of a statement and establish its truth value G.G.26 Identify and write the inverse, converse, and contrapositive of a given conditional statement and note the logical equivalences G.G.27 Write a proof arguing from a given hypothesis to a given conclusion G.G.33 Investigate, justify, and apply the triangle inequality theorem G.G.34 Determine either the longest side of a triangle given the three angle measures or the largest angle given the lengths of three sides of a triangle G.G.42 Investigate, justify, and apply theorems about geometric relationships, based on the properties of the line segment joining the midpoints of two sides of the triangle

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Mathematics Scope and Sequence – High School
Geometry

September / October	October / November	December / January
Unit 1 - Introduction to Geometry: Lines, Angles, Measurement, Basis Constructions, Reasoning and Proof	***Unit 2 - Lines and Angles, Congruent Triangles***	***Unit 3 - Relationship within Triangles, Similarities***
Coordinate Geometry G.G.66 Find the midpoint of a line segment, given its endpoints G.G.67 Find the length of a line segment, given its endpoints G.G.69 Investigate, justify, and apply the properties of triangles and quadrilaterals in the coordinate plane, using the distance, midpoint, and slope formulas	G.G.32 Investigate, justify, and apply theorems about geometric inequalities, using the exterior angle theorem G.G.35 Determine if two lines cut by a transversal are parallel, based on the measure of given pairs of angles formed by the transversal and the lines G.G.36 Investigate, justify, and apply theorems about the sum of the measures of the interior and exterior angles of polygons ***Transformational Geometry*** G.G.63 Determine whether two lines are parallel, perpendicular, or neither, given their equations G.G.64 Find the equation of a line, given a point on the line and the equation of a line perpendicular to the given line G.G.65 Find the equation of a line, given a point on the line and the equation of a line parallel to the desired line ***Coordinate Geometry*** G.G.62 Find the slope of a perpendicular line, given the equation of a line G.G.63 Determine whether two lines are parallel, perpendicular, or neither, given their equations G.G.64 Find the equation of a line, given a point on the line and the equation of a line perpendicular to the given line G.G.65 Find the equation of a line, given a point on the line and the equation of a line parallel to the desired line	G.G.43 Investigate, justify, and apply theorems about the centroid of a triangle, dividing each median into segments whose lengths are in the ratio 2:1 G.G.44 Establish similarity of triangles, using the following theorems: AA, SAS, and SSS G.G.45 Investigate, justify, and apply theorems about similar triangles G.G.46 Investigate, justify, and apply theorems about proportional relationships among the segments of the sides of the triangle, given one or more lines parallel to one side of a triangle and intersecting the other two sides of the triangle ***Transformational Geometry*** G.G.58 Define, investigate, justify, and apply similarities (dilations and the composition of dilations and isometries)

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Mathematics Scope and Sequence – High School
Geometry

February	March	April / May	May / June
Unit 4 - Right Triangle and Trigonometry, Quadrilaterals	***Unit 5 - Transformations***	***Unit 6 - Area, Surface Area, Volume***	***Unit 7 - Circles***
Formal and Informal Proofs G.G.36 Investigate, justify, and apply theorems about the sum of the measures of the interior and exterior angles of polygons G.G.38 Investigate, justify, and apply theorems about parallelograms involving their angles, sides, and diagonals G.G.39 Investigate, justify, and apply theorems about special parallelograms (rectangles, rhombuses, squares) involving their angles, sides, and diagonals G.G.40 Investigate, justify, and apply theorems about trapezoids (including isosceles trapezoids) involving their angles, sides, medians, and diagonals G.G.41 Justify that some quadrilaterals are parallelograms, rhombuses, rectangles, squares, or trapezoids G.G.47 Investigate, justify, and apply theorems about mean proportionality - the altitude to the hypotenuse of a right triangle is the mean proportional between the two segments along the hypotenuse - the altitude to the hypotenuse of a right triangle divides the hypotenuse so that either leg of the right triangle is the mean proportional between the hypotenuse and segment of the hypotenuse adjacent to that	***Transformational Geometry*** G.G.54 Define, investigate, justify, and apply isometries in the plane (rotations, reflections, translations, glide reflections) *Note: Use proper function notation* G.G.55 Investigate, justify, and apply the properties that remain invariant under translations, rotations, reflections, and glide reflections G.G.56 Identify specific isometries by observing orientation, numbers of invariant points, and/or parallelism G.G.57 Justify geometric relationships (perpendicularity, parallelism, congruence) using transformational techniques (translations, rotations, reflections) G.G.58 Define, investigate, justify, and apply similarities (dilations and the composition of dilations and isometries) G.G.59 Investigate, justify, and apply the properties that remain	***Geometric Relationships*** G.G.10 Know and apply that the lateral edges of a prism are congruent and parallel G.G.11 Know and apply that two prisms have equal volumes if their bases have equal areas and their altitudes are equal G.G.13 Apply the properties of a regular pyramid, including: • lateral edges are congruent • lateral faces are congruent isosceles triangles • volume of a pyramid equals one-third the product of the area of the base and the altitude. G.G.14 Apply the properties of a regular cylinder, including: • bases are congruent • volume equals the product of the area of the base and the altitude • lateral area of a right circular cylinder equals the product of an altitude and the circumference of the base G.G.15 Apply the properties of a right circular cone, including: • lateral area equals one-half the product of the slant height and the	***Coordinate Geometry*** G.G.49 Investigate, justify, and apply theorems regarding chords of a circle: • perpendicular bisectors of chords • the relative lengths of chords as compared to their distance from the center of the circle G.G.50 Investigate, justify, and apply theorems about tangent lines to a circle: • a perpendicular to the tangent at the point of tangency • two tangents to a circle from the same external point • common tangents of two non-intersecting or tangent circles G.G.51 Investigate, justify, and apply theorems about the arcs determined by the rays of angles formed by two lines intersecting a circle when the vertex is: • inside the circle (two chords) • on the circle (tangent and chord) • outside the circle (two tangents, two secants, or tangent and secant) G.G.52 Investigate, justify, and apply theorems about arcs of a circle cut by two parallel lines G.G.53 Investigate, justify, and apply theorems regarding segments intersected by a circle:

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Mathematics Scope and Sequence – High School
Geometry

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leg G.G.48 Investigate, justify, and apply the Pythagorean theorem and its converse ***Coordinate Geometry*** G.G.66 Find the midpoint of a line segment, given its endpoints G.G.67 Find the length of a line segment, given its endpoints	invariant under similarities G.G.60 Identify specific similarities by observing orientation, numbers of invariant points, and/or parallelism G.G.61 Investigate, justify, and apply the analytical representations for translations, rotations about the origin of 90º and 180º, reflections over the lines $x = 0$, $y = 0$, and $y = x$, and dilations centered at the origin	circumference of its base • volume is one-third the product of the area of its base and its altitude G.G.16 Apply the properties of a sphere, including: • the intersection of a plane and a sphere is a circle • a great circle is the largest circle that can be drawn on a sphere • two planes equidistant from the center of the sphere and intersecting the sphere do so in congruent circles • surface area is $4\pi r^2$ • volume is $\frac{4}{3}\pi r^3$ ***Constructions*** G.G.20 Construct an equilateral triangle, using a straightedge and compass, and justify the construction ***Locus*** G.G.27 Write a proof arguing from a given hypothesis to a given conclusion.	• along two tangents from the same external point, • along two secants from the same external point, • along a tangent and a secant from the same external point, • along two intersecting chords of a given circle G.G.71 Write the equation of a circle, given its center and radius or given the endpoints of a diameter G.G.73 Find the center and radius of a circle, given the equation of the circle in center-radius form G.G.74 Graph circles of the form $(x-h)^2 + (j-k)^2 = r^2$