

Introduction to Mathematics Scope and Sequence

Although educators in every subject area want to improve their students' performance, the need to do so in mathematics is particularly urgent, based on the results of recent international comparisons.

In 2003, U.S. students' mathematics performance was assessed by the Trends in International Mathematics and Science Study (TIMSS) for students in grades 4 and 8 and by the Program for International Student Assessment (PISA) for students at age 15. To address the issue of making meaningful interpretations of U.S. performance relative to other countries, the American Institutes for Research identified the 12 industrialized countries that participated in all three assessments. Along with the United States, the other 11 countries are Australia, Belgium, Hong Kong, Hungary, Italy, Japan, Latvia, Netherlands, New Zealand, Norway, and the Russian Federation. These 12 countries span four continents—North America, Australia/Oceania, Asia, and Europe—and constitute a broad range of industrialized nations for comparison. As the chart below indicates, the United States ranks 8th on TIMSS-4, 9th on TIMSS-8, and 9th on PISA, age 15. Hence, the comparison changes little as U.S. students progress through school—a consistent picture

Exhibit 2. Scores and Rankings² of 12 Countries Participating on the 2003 International Mathematics Assessments: TIMSS Grades 4 and 8, and PISA Age 15¹

Country	TIMSS Grade 4		TIMSS Grade 8		PISA Age 15	
	Score	Rank	Score	Rank	Score	Rank
AUS	499*	10	505	8	524*	5
BEL	551*	3	537*	3	529*	4
HKG	575*	1	586*	1	550*	1
HUN	529*	7	529*	5	490	8
ITL	503*	9	484*	11	466*	12
JPN	565*	2	570*	2	534*	3
LAT	536*	5	508	6	483	9
NLD	540*	4	536*	4	538*	2
NZL	493*	11	494*	10	523*	6
NOR	451*	12	461*	12	495*	7
RUS	532*	6	508	6	468*	11
USA	518	8	504	9	483	9
AVG	524		519		507	
	Countries statistically above U.S. = 7 Countries statistically below U.S. = 4 Difference = 3 countries statistically above U.S.		Countries statistically above U.S. = 5 Countries statistically below U.S. = 3 Difference = 2 countries statistically above U.S.		Countries statistically above U.S. = 6 Countries statistically below U.S. = 3 Difference = 3 countries statistically above U.S.	
	Scorecor (TIMSS4,TIMSS8) = .93*		Rnkcor (TIMSS4,TIMSS8) = .96*			
	Scorecor (TIMSS8,PISA15) = .87*		Rnkcor (TIMSS8,PISA15) = .66*			
¹ Indicates country scored statistically significant above or below the United States at the .05 level. ² Country rankings are from highest score (equals 1) to lowest score (equals 12). ³ Tunisia also participated in all three international results, but it is not an industrialized country and was omitted from our study. Source: Mullis, Martin, Gonzalez, and Chrostowski, 2004; OECD, 2004.						

of overall mediocrity. For the complete AIR analysis, please visit:
http://www.air.org/news/documents/TIMSS_PISA%20math%20study.pdf

Such results have spurred a nationwide effort to improve the ways that mathematics is taught and learned in our schools. Led by the National Council of Teachers of Mathematics (NCTM), states began to adopt learning standards in the 1990s and that trend has accelerated in the new century. The New York State Learning Standards are a direct result of, and are based on, the pioneering work done by NCTM. For a detailed look at NCTM's Performance Standards visit
<http://standards.nctm.org/document/appendix/numb.htm>

These standards are comprehensive, and in response to frequently-asked questions from around the country, in 2006 NCTM produced a further refinement: its Curriculum Focal Points. These provide descriptions of the most significant mathematical concepts and skills at each grade level and were chosen according to three criteria: they are mathematically important, both for further study in mathematics and for use in applications in and outside of school; they conform to what is known about learning mathematics; they connect logically with the mathematics in earlier and later grade levels. For more information about the Curriculum Focal Points visit

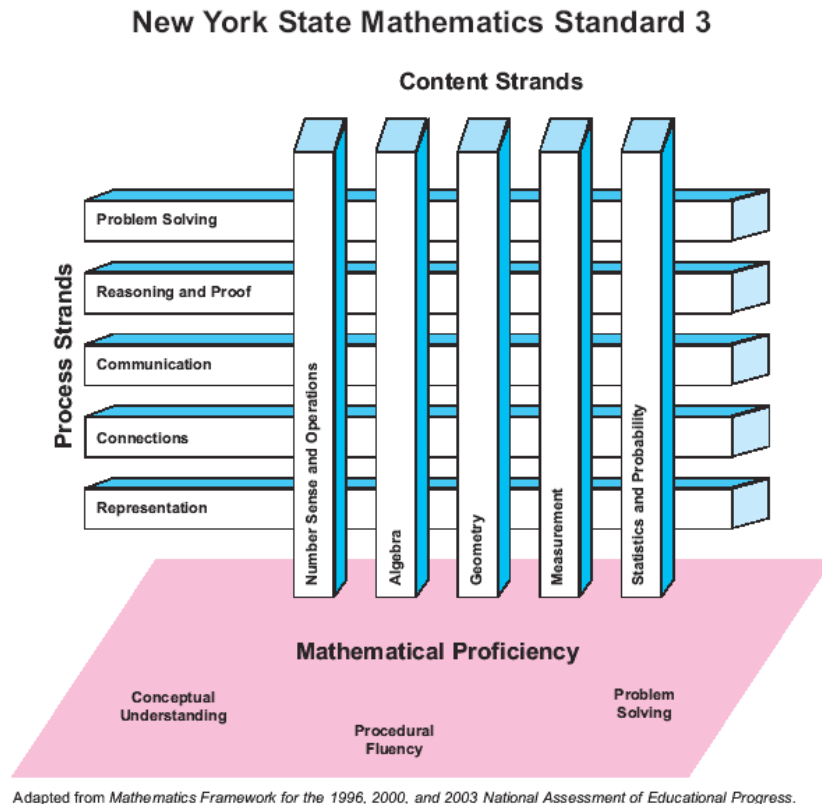
<http://www.nctm.org/standards/content.aspx?id=270&linkidentifier=id&itemid=270>

We have incorporated the relevant focal points in our preparation of the Scope and Sequence for each grade. In addition, we have been guided by recommendations from the U.S. President's National Mathematics Advisory Panel, which released its report in March 2008, in particular its first recommendation, "A focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics, should become the norm in elementary and middle school mathematics curricula." The full report is available at <http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf>

Mathematics Scope and Sequence documents offer a grade-by-grade overview of all the topics which students are expected to know, from Kindergarten through grade 10. Topics are performance indicators taken from the New York State Learning Standards for Mathematics (2005) and within each year are listed chronologically, to provide a sense of their development over time including, for grades 3 through 8, pre-May and post-May assessment items. (Since the yearly assessments for each grade are administered in May, topics taught after that month will be assessed the following year; for example, grade 5 performance indicators taught in May and June are assessed on the grade 6 exam). In addition, performance indicators are separated into five content strands: Number Sense and Operations, Algebra, Geometry, Measurement and Statistics and Probability, which provide a broader view of **what** students should be learning each year.

However, the performance indicators should not be viewed as a checklist of skills void of understanding and application. In fact, by describing **how** students should learn, process strands (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation) highlight ways of acquiring and using content knowledge and help students to see mathematics as a discipline rather than a set of isolated skills. Student engagement in mathematical content is accomplished through these strands, which are

integrated throughout the scope of the curriculum. A graphic representation of this integration is presented below.



For a complete listing of both content and process strands, visit:
<http://www.emsc.nysed.gov/3-8/MathCore.pdf>

Effective instructional strategies and strategies to address the needs of all learners, including ELL students, students with special needs and enrichment are addressed through the core curriculum chosen for each grade level, e.g., *Everyday Math* in elementary school, *Impact Math* in middle school and *Prentice Hall New York Integrated Algebra* in ninth grade. We have provided a link to the Daily Planning Guide for the relevant curriculum in the Scope and Sequence for each grade. Access to all of the Planning Guides is available here:
<http://schools.nyc.gov/Academics/Mathematics/EducatorResources/PacingPlanningGuides>
Links to other core curriculum resources can be found on the DOE math website at:
<http://schools.nyc.gov/Academics/Mathematics/EducatorResources/CCOnlineModules.htm>

Schools which are not using the core curriculum must develop or identify resources to ensure that the curriculum which they are using is aligned to the NY State Standards and addresses all performance indicators for the grade or course, including pre- and post-March items. For example their grade-8 curriculum must address post-March topics from grade 7.

For such schools, curriculum mapping would also be necessary in order to develop a pacing and planning calendar. In addition, schools would have to either identify programs that have resources to support differentiation or develop their own.

Research-based strategies which we recommend to support all learners are visual and graphic depiction of problems, student think-alouds, explicit instruction, peer-assisted learning and formative assessments. A valuable meta-analysis of such strategies has been done by the Center on Instruction; more detail on this study is available at:

<http://centeroninstruction.org/files/Russell%20Gersten%20David%20Chard%20Effective%20Instruction1.pdf>

In addition, strategies to introduce, reinforce and expand student literacy and use of academic language are encouraged at every grade level. A comprehensive description of resources is available at:

<http://schools.nyc.gov/StudentSupport/AcademicInterventionServices/EducatorResources/default.htm> and more activities and materials can be found at:

http://www.austinschools.org/matrix/0607/Math/el/resources/Math_Elem_Vocab_Packet_06.pdf Educators may also refer to glossaries published by the NY State Education Department for elementary and middle schools at: <http://www.emsc.nysed.gov/3-8/pk-8glossary.doc> and for high school at: <http://www.emsc.nysed.gov/3-8/glossaryHS.doc>

Of course language development is also a major focus of our work with English Language Learners. Bilingual glossaries are an important tool in our efforts and may be accessed at: http://www.emsc.nysed.gov/biling/bilingual_glossaries.htm

Throughout our curriculum, a major emphasis is placed on enrichment for students at every grade level. For example, a majority of our middle schools now offer acceleration for eighth-graders who wish to participate in high-school level courses i.e., Integrated Algebra. Such students must complete coursework in order to sit for the Regents exam and pass the exam in order to receive high school credit for the course, as per the NY State Education Department's Part 100 regulations:

<http://www.emsc.nysed.gov/part100/pages/1004.html#d>

The pacing calendar for the accelerated 8th grade math curriculum which will prepare students to take both the NYS 8th grade math test and the Integrated Algebra curriculum can be found on the DOE math website at:

http://schools.nycenet.edu/offices/teachlearn/mathematics/Grade8_Acc_Algebra.pdf

Enrichment for students also takes the form of participation in special projects, events and competitions, such as

- First Robotics <http://www.usfirst.org/>
- Future Cities www.futurecity.org
- First in Math (Game of 24) <http://www.firstinmath.com/>
- Math Teams
- Moody's Mega Math Challenge <http://m3challenge.siam.org/>

- Math Olympiads <http://www.moems.org/>

The work being asked of youngsters in these Scope and Sequence documents can be organized into four broad categories which build on one another: knowing facts and procedures, using concepts, solving problems, and mathematical reasoning. These categories are ordered so that in general, the cognitive complexity of tasks increases from one broad cognitive domain to the next. The less demanding end of the cognitive spectrum, knowing facts and procedures, involves recalling definitions, recognizing mathematically equivalent entities, and correctly performing computational procedures. At the other end, reasoning requires students to analyze more complex problems by breaking them into their parts, making connections between different mathematical ideas, or solving non-routine problems that they are not likely to have seen before. (*American Institutes for Research*, 2005). Our goal is to move our youngsters to this highest level of mathematical rigor.

What follows are characteristics of a standards-based math classroom:

Nature of Instruction

- challenges students with intellectually and academically rigorous instruction;
- reflects a core of knowledge that is important, aligned to standards and organized around major concepts and skills;
- makes interdisciplinary connections between and among subjects;
- gives students opportunities to engage in tasks that are experiential and authentic in meaningful contexts;
- provides students with an opportunity to evaluate and revise work in collaboration with peers and teachers so that the work ultimately meets standards;
- engages students individually, in groups, and as a whole class or community;
- involves the use of a variety of resources and appropriate materials available within and beyond the classroom;
- engages students in accountable talk and high level thinking; requires them to raise questions, solve problems, and apply reason to a problem/task;
- requires teachers to be learners who know and understand content and underlying concepts.

The Role of the Teacher

- provides quality instruction that includes a variety of strategies and opportunities adapted to diverse learners;
- selects learning tasks with specific goals in mind;
- establishes clear expectations for meeting or exceeding standards;
- meets with students regularly to help them assess their progress toward meeting standards;
- establishes an appropriate classroom climate that supports active learning, interaction, and self-motivation.

Classroom Climate

- provides an attractive, vibrant, dynamic learning environment that reflects work in progress, and appreciation of the social nature of learning, and appropriate materials that support learning;
- reflects public sharing and frequent recognition of student work and accomplishments;

- promotes positive teacher-student and student-student interaction in an environment of respect;
- reflects cooperative, collaborative activities that form the class into an interdependent community;
- reflects validation of every student's language, culture, and background; reflects, nurtures, and supports a spirit of inquiry, thoughtful questioning, and critical thinking;
- encourages the effective and appropriate use of time on task.

Role of the Student

- engages in tasks as an active learner who takes full advantage of opportunities to learn;
- shares accountability with teachers as he/she strives to attain standards;
- voices interests and concerns that help shape classroom investigations;
- interacts with teachers and peers in positive and constructive ways that enhance the sense of community within the classroom;
- engages in thoughtful reflection and assessment, with the support of teachers about his/her progress toward standards;
- shares the belief that sincere and consistent personal effort over time will serve to increase his/her ability.

Authentic and Reflective Assessment

- references fair and credible evaluations to the standards;
- includes multiple techniques that are understood and articulated by students;
- informs and shapes instruction based on inquiry as an ongoing process.

Access, Equity, and Excellence

- considers all students in the planning and choosing of learning tasks;
- encourages every student to contribute and values every student's contribution;
- gives every student the opportunity to work with appropriate materials that support learning experiences;
- gives every student sufficient time and appropriate support for learning.

The process of moving toward the standards-based classroom characterized above takes time and effort. To help educators assess their progress, the *Stages of Implementation* document can be a useful tool:

<http://schools.nyc.gov/NR/ronlyres/B3699A0A-CCCF-42FF-A7D3-F69ABF7AE55E/54690/StagesofImplementationMath.pdf>