The van Hiele Framework

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The cognitive- and spatial-development processes of geometry may be framed within a theoretical construct developed by Dina van Hiele and Pierre van Hiele, two mathematics educators, in the Netherlands during the 1950s and 1960s. Concerned about the difficulties their middle-grades students were having in geometry, the van Hieles began thinking that the content they were introducing was too advanced for many of their students to fully understand. This assumption led them to explore the prerequisite reasoning abilities needed for successfully engaging in a logical-deductive system of thought. They described five levels of thought that characterize the thinking in which children engage as they become more sophisticated in their understanding of geometric relationships:

Level 0: Visual, in which the student identifies, names, compares, and operates on geometric shapes

Level 1: Analysis, in which the student analyzes the attributes of shapes and the relationships among the attributes of shapes and discovers properties and rules through observation

Level 2: Informal deduction, in which the student discovers and formulates generalizations about previously learned properties and rules and develops informal arguments to show those generalizations to be true

Level 3: Deduction, in which the student proves theorems deductively and understands the structure of the geometric system

Level 4: Rigor, in which the student establishes theorems in different systems of postulates and compares and analyzes deductive systems (Malloy [1999], as adapted from Fuys, Geddes, and Tischler [1988] and O'Daffer and Clemens [1992])

These cognitive and spatial levels are important to teachers in the middle grades. *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM] 2000) recommends standards for school mathematics in the middle grades and suggests expectations for middle-grades students that are more ambitious than many curricula have been designed to meet. The van Hiele framework suggests strategies that can help students clarify and understand geometric concepts. This framework is offered as one way of thinking about students' growth in geometric understanding.

In an ideal world, students from prekindergarten through high school would learn to think and reason about geometry in a similar progression. Students in prekindergarten through grade 2 would focus on the visualization level, students in grades 2–5 on the analysis level, students in grades 5–8 on the informal-deduction level, and high school students on the deduction level. However, this is not always the case. By the middle grades, many students' cognitive and spatial development appears to be between the visual (level 0) and informal-deduction

Those who want to learn more about the van Hiele model can consult The van Hiele Model of Thinking in Geometry among Adolescents *(Fuys, Geddes, and Tischler 1988) and*

"The van Hiele Model of the Development of Geometric Thought" (Crowley 1987). The latter can be found on the CD-ROM.

(level 2) levels as defined by the van Hieles. Therefore, the discussion of teaching strategies in *Navigating through Geometry in Grades 6–8* focuses primarily on visualization, analysis, and informal deduction (levels 0–2). Because a few middle-grades students may be at level 3 and because middle-grades students should be progressing toward this level, the discussion also includes some work at level 3, which is typically the instructional level of high school geometry courses.

In implementing instruction based on the van Hiele framework, teachers have two tasks. First, teachers need to recognize and understand the van Hiele levels of their students, and second, they need to help their students progress through these levels in preparation for the axiomatic, deductive reasoning that is required in high school geometry. As an example, consider the use of the van Hiele levels to explain the understanding and actions of students as they try to conceptualize similarity and use the concept (Fuys, Geddes, and Tischler 1988; Van de Walle 1998). This example involves similarity of triangles.

Level 0. Students are able to identify and group shapes into categories or classes simply on the basis of how the shapes are alike. They can recognize and name triangles on the basis of their visual characteristics. Given two similar triangles, they can identify them as two triangles because they "look alike." These students can tell that one triangle is bigger or that one is smaller, but they will *not* conceptualize the properties of similarity. They can identify parts of the triangles and may even line up the two triangles to see that the angles are the same size, but they do *not* analyze a figure according to its components. The appearance of a shape defines what it is and overpowers attention to the properties of that shape.

Level 1. Students are capable of thinking about the properties of shapes. They can separate right, obtuse, and acute triangles into different classes of triangles. They can identify triangles that are equilateral as having three congruent sides and isosceles triangles as having two congruent sides. Given two similar triangles, these students can identify them as having the same shape. Going further, they can compare the sizes of the angles and match the corresponding angles in the two triangles. They can also measure the sides to see that the corresponding sides of similar triangles are proportional. Level 1 students are able to separate sets of similar triangles into classes, understanding that all triangles are not similar and that there are different sets of similar triangles (see fig. 1). These students can compare triangles according to their relationships, but they *cannot* generalize how properties are interrelated; they cannot, for instance, reason that if corresponding angles are congruent, then corresponding sides are proportional.

Level 2. Students are able to think about relationships among properties of classes of shapes. Given any pair of similar triangles, they can understand that similarity results in corresponding angles being congruent and corresponding sides being proportional. Moreover, these students are able to develop an understanding of relationships between and among properties and demonstrate a greater ability to apply "if-then" reasoning. They can conclude that if two triangles are similar, then the corresponding angles are congruent. They also can conclude

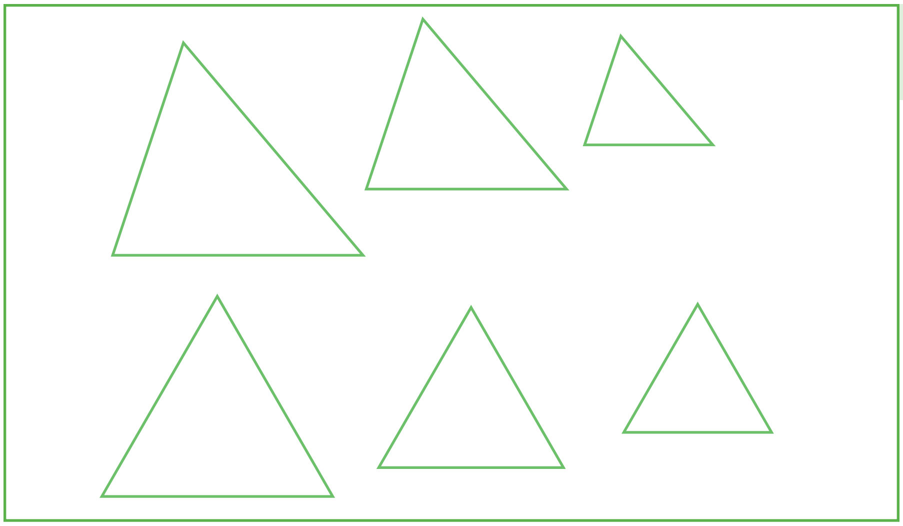

Fig. **1.**

Two sets of similar triangles

that if corresponding angles in two triangles are congruent, then the corresponding sides are proportional. They are *not* able to distinguish between necessary and sufficient conditions related to similarity. These students can construct informal arguments to show that equilateral triangles are always similar and that isosceles triangles are sometimes similar. However, these students are not ready for the axiomatic structures of deductive reasoning.

Level 3. Students can extend their examination of the properties of shapes. They recognize the need for axioms, postulates, and theorems that clarify a system of properties. Students at this level are able to reason about similarity of triangles abstractly through formulizing deductive arguments that reach logical conclusions; they can, for example, develop proofs from axioms and theorems. They can also understand and demonstrate the necessary and sufficient conditions for similarity.

This brief summary of how students think about geometry reveals that students at different van Hiele levels operate geometrically in distinct ways—the objects of their thought differ, and therefore they learn and progress to higher levels of geometric thinking through different instructional strategies. An understanding of the geometric focus of students' thoughts at each van Hiele level is important for the development of appropriate instruction, language, and learning tools. This framework gives mathematics educators insights about the ways in which instructional decisions can be made to promote students' growth and development in spatial and geometric reasoning.

One of the strengths of using the van Hiele framework to do so is that students' progression from one level to the next depends more on instruction than on the age or maturation of the student. In the middle grades, five instructional phases can be applied to teaching geometry (Fuys, Geddes, and Tischler 1988): First, students gather information by working with examples and nonexamples of concepts. Second, they are provided appropriate information and complete tasks that develop relationships. Third, they become aware of the relationships and explain them using appropriate geometric language. Fourth, the students complete additional, more complex tasks to build their understanding of the relationships. Finally, they summarize what they have learned and reflect on it. The five phases occur within each level as students move from one level to the next.

As students work their way through the levels in succession, their instruction should include geometric language, tools, symbols, and relationships appropriate to the subject or topic being taught. Because students in the middle grades are at different stages of development, they should have the opportunity to work with concrete tools, drawings, and symbolic notation. The activities and illustrations in *Navigating through Geometry in Grades 6–8* were selected to assist teachers in providing the types of experiences that allow students to attain level 2 of the van Hiele framework.

References

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